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NONPARAMETRIC METHODS WITH APPLICATIONS TO RELIABILITY.(U)
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During the period (October 1, 1976 through September 30, 1978) under review, the principle investigator has been mainly interested in the development of various methods in nonparametric and Bayesian inference with applications that are particularly relevant to reliability studies. Twelve technical reports were produced under this grant, and six articles were published in technical journals.

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LEVEL II

Summary of Accomplishments on AFOSR 76-3109

"Nonparametric Methods with Applications to Reliability"

Principal Investigator: Myles Hollander

Department of Statistics, Florida State University

Grant Period: October 1, 1976 through September 30, 1978

Date of Summary: September, 1978

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Preface

This is the summary report for AFOSR grant 76-3109 "Nonparametric Methods with Applications to Reliability." The grant period was October 1, 1976 through September 30, 1978. [Beginning October 1, 1978, the principal investigator Myles Hollander continues research as co-principal investigator (with Frank Proschan) under AFOSR grant 76-3678 "Statistical Aspects of Reliability, Maintainability, and Availability."]

In this summary we:

a. List specifically the accomplishments in terms of technical reports that have been written, and articles that have been published or are scheduled to appear.

b. Give, for each paper and/or technical report, a technical summary (abstract), and a non-technical summary with the latter describing potential applications to the Air Force.

c. List the specific activities and honors of note of the principal investigator during the contract period.

1. List of Accomplishments

Technical Reports Produced Under Grant AFOSR 76-3109

1. Nonparametric empirical Bayes estimation of the probability that $X \leq Y$, by M. Hollander and R. M. Korwar, #1, October, 1976.

2. Testing for agreement between two groups of judges, by M. Hollander and J. Sethuraman, #2, January, 1977.

3. Nonparametric Bayes estimation with incomplete Dirichlet prior information, by G. Campbell and M. Hollander, #3, June, 1977.

4. Prediction intervals with the Dirichlet prior, by G. Campbell and M. Hollander, #4, June, 1977.

5. Testing whether more failures occur later, by M. Hollander, #5, October, 1977.

6. Testing to determine the underlying distribution using randomly censored data, by M. Hollander and F. Proschan, #6, May, 1978.

7. On the normal convergence of a family of simple epidemics, by N. A. Langberg, #7, June, 1978.

8. The discrete asymptotic behaviour of a simple batch epidemic process, by L. Billard, H. Lacayo, and N. Langberg, #8, June, 1978.

9. On the negative binomial convergence in a class of m-dimensional epidemics, by H. Lacayo and N. Langberg, #9, July, 1978.

10. The symmetric m-dimensional simple epidemic process, by L. Billard, H. Lacayo, and N. Langberg, #10, June, 1978.

11. Characterization of nonparametric classes of life distributions, by N. A. Langberg, R. León, and F. Proschan, #11, July, 1978.

12. On a characterization of multivariate distributions with applications in reliability and epidemiology, by N. A. Langberg, #12, August, 1978.

Articles and Books (Published or In Press)

1. Nonparametric empirical Bayes estimation ofthe probability that $X \leq Y$, by M. Hollander and R. M. Korwar. Communications in Statistics - Theor. Meth., A5(14), 1369-1383, 1976.

2. Statistics: A Biomedical Introduction by B. W. Brown, Jr. and M. Hollander. A volume in the Wiley Series in Probability and Statistics, John Wiley, New York, 1977.

3. Functions decreasing in transposition and their applications in ranking problems, by M. Hollander, F. Proschan, and J. Sethuraman. Annals of Statistics 5 722-733, 1977.

4. Rank order estimation with the Dirichlet prior, by G. Campbell and M. Hollander. Annals of Statistics 6, 142-153, 1978.

5*. Nonparametric estimation of distribution functions, by M. Hollander and R. M. Korwar. The Theory and Applications of Reliability, Vol. I., ed. by C. P. Tsokos and I. N. Shimi, Academic Press, New York, 85-107, 1977.

6. Testing whether more failures occur later, by M. Hollander. Proceedings of the 1978 Annual Reliability and Maintainability Symposium, 103-106, 1978.

7. Testing for agreement between two groups of judges, by M. Hollander and J. Sethuraman. To appear in Biometrika 65, 1978.

8. Nonparametric Bayes estimation with incomplete Dirichlet prior information, by G. Campbell and M. Hollander. To appear in Optimizing Methods in Statistics-II, ed. by J. Rustagi, Academic Press, New York, 1979.

* This paper was not written or revised during the contract period and thus it does not acknowledge Grant 76-3109. It is included here for completeness since it is a publication of the principal investigator which appeared during the contract period.

2. Discussion of papers.

2.1. Nonparametric empirical Bayes estimation of the probability that $X \le Y$.

2.1.a. Abstract.

Let X and Y be two real valued independent random variables with distribution functions F and G, respectively. We consider the problem of estimating the probability that $X \le Y$, denoted by Δ, $\Delta = \int F dG$. Motivated by Ferguson's (1973) nonparametric Bayes estimator, we propose an empirical Bayes estimator of Δ which requires less prior information about $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$, the parameters of the (assumed) underlying Dirichlet processes. Only $\alpha_1(R)$ and $\alpha_2(R)$ need be specified. Consider, then, the following set up appropriate for an empirical Bayes estimation problem. Let $\{X_i^{(1)}\}$, $\{X_i^{(2)}\}$, $i = 1,2,\ldots$, be two independent sequences of independent vectors of observations from F and G respectively. Here $X_i^{(j)} = (X_{i1}^{(j)},\ldots,X_{in_j}^{(j)})$, $j = 1,2$ and $i = 1,2,\ldots$. Assume independent Dirichlet priors on (R, B) with parameters α_1 and α_2 respectively for F and G. Here R is the real line and β is the σ-field of Borel subsets of R. Let the action space be the closed interval $[0, 1]$, and the loss function be $L(\Delta, \hat{\Delta}) = (\Delta - \hat{\Delta})^2$, where $\hat{\Delta}$ is an estimator of Δ. We assume $\alpha_1(R)$ and $\alpha_2(R)$ are known. We then propose the estimator $\hat{\Delta}_n$ below as an estimator of Δ on the (n+1)th occasion. The estimator is given by

$$\hat{\Delta}_n = \hat{\Delta}(X_1^{(1)},\ldots,X_{n+1}^{(1)};\ X_1^{(2)},\ldots,X_{n+1}^{(2)})$$

$$= p_{1,n_1} p_{2,n_2} \frac{1}{n^2 n_1 n_2} \sum_{i=1}^{n} \sum_{j=1}^{n_2} \sum_{k=1}^{n} \sum_{\ell=1}^{n_1} I_{(-\infty, X_{ij}^{(2)}]}(X_{k\ell}^{(1)}) \qquad (1)$$

$$+ p_{1,n_1}(1-p_{2,n_2}) \frac{1}{n n_1 n_2} \sum_{j=1}^{n_2} \sum_{k=1}^{n} \sum_{\ell=1}^{n_1} \delta_{X_{k\ell}^{(1)}}((-\infty, X_{n+1,j}^{(2)}])$$

$$+ (1 - p_{1,n_1}) p_{2,n_2} \frac{1}{n_1} \sum_{i=1}^{n_1} \left\{ 1 - \frac{1}{nn_2} \sum_{j=1}^{n} \sum_{k=1}^{n_2} \delta_{X_{jk}^{(2)}} ((-\infty, X_{n+1,i}^{(1)})) \right\}$$

$$+ (1 - p_{1,n_1})(1 - p_{2,n_2}) \frac{1}{n_1 n_2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} I_{(-\infty, X_{n+1,j}^{(2)}]}(X_{n+1,i}^{(1)}),$$

where $p_{i,n_i} = \alpha_i(R)/\{\alpha_i(R) + n_i\}$, $i = 1, 2$, and

$$\delta_x(A) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

2.1.b. Potential use to Air Force.

Estimation of $P(X < Y)$, the probability that X is less that Y, is particularly important in engineering situations where X is stress and Y is strength. Applications abound in materials development and evaluation situations. For example, are particular nosetips and heatshields of reentry vehicles sufficiently strong to withstand erosion due to impact of rain, ice, debris, etc.? Here the stress is created partially by the thermal environment. The estimator given by (1) enables effective utilization of available data (present and past samples) without the imposition of overly restrictive distributional assumptions.

2.1.c. Status of paper.

This has appeared as an invited paper in a special issue (December, 1976), of Communications in Statistics, devoted to recent advances in nonparametric methods.

2.2. Testing for agreement between two groups of judges.

2.2.a. Abstract.

The "problem of m rankings", so named by Kendall and studied extensively by Kendall and Babington Smith (1939), Kendall (1970), and others, considers the relationship between the rankings that a group of m judges assigns to a set of k objects. In this paper we suppose there are two groups of judges ranking the objects, and we address the following question (posed to us by Kendall): Given that there is agreement within each group of judges, how can one test for evidence of agreement between the two groups? A test of agreement proposed by Schucany and Frawley (1973), and further advanced by Li and Schucany (1975), is shown to be misleading and does not provide a satisfactory answer to Kendall's question. We then adapt a procedure, proposed by Wald and Wolfowitz (1944) in a slightly different context, to furnish a new test for agreement between two groups of judges.

2.2.b. Potential use to Air Force.

A team of m experts at Aerospace Laboratory 1 ranks k = 4 escape systems. Rank 1 is assigned to the most preferred escape system, rank to the second preferred, and so forth, so that rank 4 is assigned to the least preferred. A different team of experts ranks the same 4 escape systems at Aerospace Lab 2. It is found that there is general agreement *within* each team, and it is then desired to test for agreement *between* teams. This can be done by the distribution-free test we have developed.

2.2.c. Status of paper.

This paper is scheduled to appear in Biometrika, 1978. Sir Maurice Kendall, in private correspondence, has termed the paper a major breakthrough.

2.3. Nonparametric Bayes estimation with incomplete Dirichlet prior information.

2.3.a. Abstract.

In this paper we treat the topic of incomplete information regarding the parameter α of a Dirichlet process prior. Ferguson (1973) introduced the Dirichlet process for the incorporation of prior information into the analysis of nonparametric problems. The process can be viewed as a prior on the set of all distributions on a measurable space $(\mathcal{X}, \mathcal{A})$. The process is parametrized by α, a non-negative, non-null finite measure on $(\mathcal{X}, \mathcal{A})$. (In this paper we restrict to situations where $\mathcal{X} = \mathcal{R}$, the real line, and $\mathcal{A} = \mathcal{B}$, the Borel σ-field.) Typically, to use estimators which are Bayes with respect to a Dirichlet process with parameter α, the statistician must provide a complete specification of the measure α. This paper develops some estimators that rely only on partial information concerning α.

One approach to incomplete information concerning α is that initiated by Doksum (1972). Doksum assumes that $\alpha(t_i, t_{i+1}]$, $i = 1,\ldots,k-1$ are known with $\alpha(\mathcal{R}-(t_1, t_k]) = 0$. That is, the values that assigns to the k-1 intervals $(t_1,t_2],\ldots,(t_{k-1},t_k]$ are known, and $\alpha(\mathcal{R}) = \alpha(t_1,t_k]$. In this paper, Doksum's technique for obtaining a mixed rule is considered and shown also to yield a G-minimax rule for a suitable choice of G. The approach is then applied to obtain a new mixed rule for estimating $Pr(X \le Y)$.

2.3.b. Potential use to Air Force.

The general approach allows one to incorporate prior information into the decision problem. Here the prior information is incomplete. In the case of complete prior information, the prior guess $\alpha(\cdot)/\alpha(\mathcal{R})$ of the underlying distribution is specified completely whereas in this approach the scientist need only "guess" or specify α on intervals. The nosetips example, discussed in 2.1.b, is applicable here since we have derived a mixed estimator for $Pr(X \le Y)$. However, the mixed estimator only requires partial prior information.

2.3.c. Status of paper.

This paper will appear in Optimizing Methods in Statistics - II, ed. by J. Rustagi, Academic Press, New York, 1979.

2.4. Prediction intervals with the Dirichlet prior.

2.4.a. Abstract.

Let $X_1,\ldots,X_n$ and $Y_1,\ldots,Y_N$ be consecutive samples from a Dirichlet process on $(\mathcal{R}, \mathcal{B})$ (the real line $\mathcal{R}$ with the Borel σ-field $\mathcal{B}$) with parameter α. Typically, prediction intervals employ the previous observations $X_1, \ldots, X_n$ in order to predict a specified function of the future sample $Y_1, \ldots, Y_N$. Here one- and two-sided prediction intervals for at least k of N future observations are developed for the situation in which, in addition to the previous sample, there is prior information available. The information is specified via the parameter α of the Dirichlet process.

2.4.b. Potential use to Air Force.

In many practical problems in quality control and statistics, one wishes to use the results of a past sample to construct an interval which will contain the results of a future sample from the same population with a specified probability. Such an interval is known as a *prediction interval*. Prediction intervals are of special interest to engineers who are concerned with setting limits on the performance of a small number of units of a product. Such limits would be required, for example, in setting specification to contain with a high probability a critical performance characteristic for all units in an order of three heavy transformers when the only available information is the data on five previous transformers of the same type. By using the limits of a prediction interval as specification limits, one is assured with a desired probability that all three transformers will meet specifications.

Typically, prediction intervals are parametric, Bayesian parametric, or (non-Bayesian) nonparametric. The prediction intervals we derive are Bayesian nonparametric, and thus simultaneously allow the user to incorporate prior infromation into the procedure, without requiring restrictive assumptions about the form of the underlying population.

2.4.c. Status of paper.

The paper has been submitted to the Journal of the American Statistical Association. It was well-received, subject to minor suggested revisions which are currently being made.

2.5. Functions decreasing in transposition and their applications in ranking problems.

2.5.a. Abstract.

Let $\underline{\lambda} = (\lambda_1,\ldots,\lambda_n)$, $\lambda_1 \leq \ldots \leq \lambda_n$, and $\underline{x} = (x_1,\ldots,x_n)$. A function $g(\underline{\lambda}, \underline{x})$ is said to be decreasing in transposition (DT) if (i) g is unchanged when the same permutation is applied to $\underline{\lambda}$ and to $\underline{x}$, and (ii) $g(\underline{\lambda}, \underline{x}) \geq g(\underline{\lambda}, \underline{x}')$ whenever $\underline{x}'$ and $\underline{x}$ differ in two coordinates only, say i and j, $(x_i - x_j)(i - j) \geq 0$, and $x_i' = x_j$, $x_j' = x_i$. The DT class of functions includes as special cases other well known classes of functions such as Schur functions, totally positive functions of order two, and positive set functions, all of which are useful in many areas including stochastic comparisons. Many well known multivariate densities have the DT property. This paper develops many of the basic properties of DT functions, derives their preservation properties under mixtures, compositions, integral transformations, etc. A number of applications are then made to problems involving rank statistics.

2.5.b. Potential use to Air Force.

The standard assumption in comparing failure times of two systems, $X_1,\ldots,X_m$ of system 1, and $Y_1,\ldots,Y_n$ of system 2, is that the X's are a random sample from one population, that the Y's are a random sample from a second population, and that all X's and Y's are mutually independent. Then to test, for example, the hypothesis of equivalent populations against the alternative that one system tends to have shorter failure times than the other, two sample rank statistics can be used. However in many Air Force applications, the assumption of independence can be violated, say by having the systems operating in a common environment. The DT paper allows for application of these rank tests even in the case of such dependency situations by providing inequalities on power functions.

2.5.c. Status of paper.

The paper appeared in the July, 1977 issue of the Annals of Statistics.

2.6. Rank order estimation with the Dirichlet prior.

2.6.a. Abstract.

Suppose that a sample of size n from a distribution function F is obtained. However, only $r(< n)$ values from the sample are observed, say $X_1,\ldots,X_r$. Without loss of generality, we can consider $X_1,\ldots,X_r$ to be the first r values in the (unordered) sample. The problem is to estimate the rank order G of X_1 among $X_1,\ldots,X_n$. The situations of interest include F non-random, either known or unknown, and F random. The random case assumes that F is a random distribution function chosen according to Ferguson's (1973) Dirichlet process prior. Since this random distribution function is discrete with probability one, average ranks are used to resolve ties. A Bayes estimator (squared-error loss) of G is developed for the random model. For the non-random distribution function model, optimal non-Bayesian estimators are developed in both the case where F is known and the case where F is unknown. These estimators are compared with the Dirichlet estimator on the basis of average mean square errors under both the random and non-random models.

2.6.b. Potential use to Air Force.

Consider for example, the following situations:

(i) An astronaut (WW, say) undergoes, as one of a pilot group of 15 astronaut trainees, extensive preparation for a space mission. Each astronaut earns a score X, a measure of overall performance. WW's score is X_1. Based on the observed values $X_1,\ldots,X_{15}$, we wish to estimate WW's rank in the total pool of 50 trainees. (Only the best ten astronauts, as measured by X, will be chosen for the mission.) Here $r = 15$ and $n = 50$.

(ii) The Mantilla River has flooded four times in this decade with the severity of each flood measured by X, the height of the river. On the basis of the observations $X_1,\ldots,X_4$, how can we estimate the severity of the first flood, in the group of these four and the next five that occur? Equivalently, how can we estimate the rank order of X_1 among $X_1,\ldots,X_9$? (Note that we could, for example, interchange the roles of X_1 and X_4 and pose the question in terms of estimating the severity of the fourth flood.) Here $r = 4$ and $n = 9$.

Of course example (ii) is easily generalized to cover other undesirable (or desirable) events, and example (i) is applicable in other situations where a subgroup is selected to be on a team or perform a mission. The rank order estimators derived can be used in these prediction problems.

2.6.c. Status of paper.

The paper appeared in the January, 1978 issue of the Annals of Statistics.

2.7. Nonparametric estimation of distribution functions.

2.7.a. Abstract.

Consider the nonparametric estimation of $n + 1$ distribution functions $F_1,F_2,\ldots,F_{n+1}$, on the basis of samples $\underline{X}_i = (X_{i1},\ldots,X_{im_i})$ from F_i, $i = 1,\ldots,n+1$. In an empirical Bayes context where $\underline{X}_1,\ldots,\underline{X}_n$ were the "past" samples, Korwar and Hollander (1976) used Ferguson's (1973) Dirichlet process to propose an estimator of F_{n+1} that utilized all $n + 1$ samples. Their estimator was developed for the equal sample sizes case $m_1 = m_2 = \ldots = m_{n+1}$. Here the Korwar-Hollander estimator is generalized to the unequal sample sizes case, and exploited for simultaneous estimation of $F_1,F_2,\ldots,F_{n+1}$. We establish a necessary and sufficient condition for the proposed estimators to be better (riskwise) than using, for each distribution, the corresponding empirical distribution function as an estimator. One sufficient condition, easily obtained from our necessary and sufficient condition, is $n \cdot \min(m_1, m_2, \ldots, m_{n+1}) > \max(m_1, m_2, \ldots, m_{n+1})$.

2.7.b. Potential use to Air Force.

In Section 4 of this paper we show how the proposed estimators can be used for the problem of simultaneously estimating n+1 distribution functions. The estimators are computed for some data of Proschan (1963) on the times of successive failures of the air conditioning systems of Boeing 720 jet airplanes. Of course, the method is applicable to survival analysis of other types of systems.

2.7.c. Status of paper.

The paper has appeared in The Theory and Applications of Reliability, Vol. I, I. N. Shimi and C. P. Tsokos editors, Academic Press, New York, 1977.

2.8. Testing whether more failures occur later.

2.8.a. Abstract.

This is an expository paper which illustrates the role of superadditivity, in reliability theory, as a means of describing wearout. Statistical inference procedures are described for testing that minus the logarithm of a life distribution is superadditive (i.e., the underlying life distribution is new better than used (NBU)), and for testing that the mean value function of a non-homogenous Poisson process is superadditive. These tests are appropriate when the underlying processes suggest that new items are better than used ones but where we do not insist that the failure rate is increasing.

2.8.b. Potential use to Air Force.

Replacement policies are essential to the maintenance of Air Force equipment. The NBU distributions play a fundamental role in the study of replacement policies. Two replacement policies commonly employed and thoroughly studied are age and block replacement. Under an *age replacement policy*, a unit is replaced upon failure or upon reaching a specified age T, whichever comes first. Under a *block replacement policy*, a replacement is made whenever a failure occurs, and additionally at specified times T,2T,3T,... . As a typical result, Marshall and Proschan show that a necessary and sufficient condition for failure-free intervals to be stochastically larger (smaller) under age replacement than under a policy of replacement at failure only is that the underlying distribution be NBU (NWU). In a similar vein, a necessary and sufficient condition that the number of failures in a specified interval [0, t] be stochastically smaller (larger) under age replacement than under a policy of replacement at failure only is that the underlying distribution be NBU (NWU). Similar comparisons hold using block replacement. Thus in reaching a decision as to whether to use an age (block) replacement policy or not, it is important to know whether the underlying distribution is NBU or not, equivalently, whether or not minus the logarithm of the life distribution is superadditive.

2.8.c. Status of paper.

This paper has appeared in the Proceedings of the 1978 Annual Reliability and Maintainability Symposium.

2.9. Testing to determine the underlying distribution using randomly censored data.

2.9.a. Abstract.

For right-censored data, we develop a goodness-of-fit procedure for testing whether the underlying distribution is a specified function G. Our test statistic C is the one-sample limit of Efron's (1967) two-sample statistic $\hat{W}$. The test based on C is compared with recently proposed competitors due to Koziol and Green (1976) and Hyde (1977). The comparisons are on the basis of (i) applicability, (ii) the extent to which the censoring distribution can affect the inference, and (iii) power. It is shown that in certain situations the C test compares favorably with the tests of Koziol-Green and Hyde.

2.9.b. Potential use to Air Force.

The need to generalize the goodness-of-fit problem to censored data arises because in some situations the observations are times to the occurrence of an end-point event and the data are analyzed before all items have experienced the event. For example, the Air Force may be testing a given component (system, etc.) to see if its life distribution can be approximated by an exponential distribution with mean 1000 days, and it may be necessary to make a decision before all of the components have failed.

2.9.c. Status of paper.

This paper has been submitted to *Biometrics*.

2.10. On the normal convergence of a family of simple epidemics.

2.10.a. Abstract.

We consider a family of simple epidemics not necessarily with exponential interinfection times. For this family of simple epidemics we establish normal approximations to all finite joint state probabilities. To illustrate the applicability of our result we present a class of simple epidemics used frequently for modeling purposes for which the normal approximations hold.

2.10.b. Potential use to Air Force.

Let a filter in an air conditioning system have N orifices that can be clogged by particles. Applying the results of the paper we can approximate the number of clogged orifices at any point in time.

2.10.c. Status of paper.

This paper has been submitted to the *Journal of Mathematical Biosciences*.

2.11. The discrete asymptotic behaviour of a simple batch epidemic process.

2.11.a. Abstract.

A simple epidemic process in which the number of individuals who can become infected at any point in time is itself a random variable is described. The discrete asymptotic behavior of such a process is discussed. In particular, the associated marginal distribution and finite-dimensional joint distribution of the limiting process are considered.

2.11.b. Potential use to Air Force.

Consider a replacement of a tire (engine, radio, etc.) of a vehicle. Although the tire is replaced the causes of failure will intensify over time. Using the results of the paper we can approximate the number of tire replacements up to time t, even when we allow more than one source to cause the failure of the tire.

2.11.c. Status of paper.

This paper has been submitted to the Journal of Applied Probability.

2.12. On the negative binomial convergence in a class of m-dimensional simple epidemics.

2.12.a. Abstract.

We consider a population which is exposed to m infections, and consists initially of N susceptibles. At each point in time at most one susceptible becomes infective, and only from one cause. This m-dimensional simple epidemic is a stochastic process, $(X_{N,1}(t),\ldots,X_{N,m}(t))$, with components counting the number of infectives from the respective causes at time t. We show that if the transition rates of cause 1 through m at time t are given by

$\alpha_i X_{N,i}(t)[1 - \frac{1}{N}\sum_{i=1}^{m}(X_{N,i}(t) - X_{N,i}(0))]$, $i = 1,\ldots,m$, $\alpha_1,\ldots,\alpha_m > 0$ and if $\lim_{N\to\infty} X_{N,i}(0) = b_i \in \{1,2,\ldots\}$, then $(X_{N,1}(t),\ldots,X_{N,m}(t))$ converges as $N \to \infty$ to a random vector with independent negative binomial components.

2.12.b. Potential use to Air Force.

Let a filter in an air conditioning system have N orifices that can be clogged by m different types of particles. Applying the results of the paper we can approximate the number of orifices clogged by the different types of particles at any point in time. This application can of course be extended to other types of systems with ventilation units, anti-pollution devices, safety latches, etc.

2.12.c. Status of paper.

This paper has been submitted to the Journal of Stochastic Processes and their Applications.

2.13. The symmetric m-dimensional simple epidemic process.

2.13.a. Abstract.

In the classical simple epidemic process [Bailey (1975)], only one disease is involved. In this work, the model is generalised to the case in which susceptibles are simultaneously exposed to m, $m \geq 1$, diseases. An individual once infected by a particular disease remains infected and is immune to further infection from another disease. The process is called an m-dimensional simple epidemic process. The limiting finite-dimensional joint state probabilities and the marginal probabilities associated with each disease are obtained.

2.13.b. Potential use to Air Force.

Let a filter in an air conditioning system have N orifices that can be clogged be m different types of particles. Applying the results of the paper we can approximate the number of orifices clogged by the different types of particles at any point in time. Again, this application can be extended to other types of systems with ventilation units, safety latches, etc.

2.13.c. Status of paper.

This paper has been submitted to The Royal Statistical Society (Series B).

2.14. Characterizations of nonparametric classes of life distributions.

2.14.a. Abstract.

In this paper we obtain characterizations of large classes of nonparametric life distributions, such as the increasing (decreasing) failure rate, increasing (decreasing) failure rate average, new better (worse) than used, etc., classes. The methods used differ from the usual functional equation methods used for the far more common characterizations of parametric families of life distributions.

2.14.b. Potential use to Air Force.

The above-mentioned classes of life distributions play central roles in the development of tests and estimators for reliability data analysis. Having new characterizations of such classes will provide new insights into developing better estimators and tests which can then in turn be used in Air Force applications.

2.14.c. Status of paper.

This paper has been submitted to the Annals of Probability.

2.15. On a characterization of multivariate distributions with applications in reliability and epidemiology.

2.15.a. Abstract.

Let $T_1,\ldots,T_n$ be positive random variables with finite means. Further let $\mathcal{I}$ be the collection of all subsets of $\{1,\ldots,n\}$, and let ξ be a function from the nth Euclidian space to $\mathcal{I}$, that equals to J, $(J \in \mathcal{I})$ at $(a_1,\ldots,a_n)$ iff $\min_{i\in J} a_iT_i < \min_{i\notin J} a_iT_i$. We prove that $\min_{1\le i\le n} a_iT_i$ and $\xi(a_1,\ldots,a_n)$ are independent random variables for every n real numbers $a_1,\ldots,a_n$ iff for every n positive real numbers $b_1,\ldots,b_n$ and $r = 1,\ldots,n$ the random variables $\min_{1\le i\le n} a_iT_i/E(\min_{1\le i\le n} a_iT_i)$ and T_r/ET_r are identically distributed. Further we provide an explicit formula for the distribution of $\xi(a_1,\ldots,a_n)$. Multivariate distributions that possess

the independence property are presented. Their use in reliability growth or decay models as well as in mathematical epidemiology are discussed.

2.15.b. Potential use to Air Force.

Using multivariate distributions presented in the paper for modeling reliability growth or decay models, will enable us to obtain estimators for the number of maintenance or decay phases that insure a certain level of reliability of the system.

2.15.c. Status of paper.

This paper has been submitted to the Journal of Multivariate Analysis.

3. Activities and honors of the principal investigator during the contract period.

In 1977 Myles Hollander received the honor of becoming an elected member of the International Statistical Institute. In April, 1978, Hollander was elected by his colleagues to become Chairman of the Department of Statistics at Florida State and on July 1, 1978 he assumed that position. Hollander's travel activities during the contract period included a March, 1977 visit to Madison, Wisconsin to lecture in the colloquia series, Department of Statistics, University of Wisconsin; a May, 1977 visit to Columbia, South Carolina to lecture in the University of South Carolina Mathematics Department's colloquia series and to give an address to the South Carolina Chapter of the American Statistical Association; a December, 1976 trip to Stanford, California to discuss potential problems in censored data analysis with Byron Wm. Brown, Jr. of Stanford University; a July, 1977 invited lecture at the Gordon Research Conference on Statistics in Chemistry (New Hampton, New Hampshire); a December, 1977 invited lecture at the 2nd International Symposium on Optimization Methods in Statistics (Bombay, India); a January, 1978 invited lecture at the Annual Reliability and Maintainability Symposium (Los Angeles, California); and an August, 1978 contributed paper presentation at the 41st annual meeting of the Institute of Mathematical Statistics.

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